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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

181. Proposed by J. F. LAWRENCE, Breckenridge, Mo.

Show that $\phi(1)\dfrac{x}{1+x^2}-\phi(3)\dfrac{x^3}{1+x^6}+\phi(5)\dfrac{x^3}{1+x^{10}}-\text{ad inf.}=\dfrac{x(1-x^2)}{(1+x^2)^2}$, $\phi(n)$ being the number of integers less than n and prime to it. [From Hall and Knight's *Higher Algebra*, page 358].

Solution* by G. B. M. ZERR, A. M., Ph. D., Parsons, West Va.

$$\phi(1)\,\frac{y}{1-y^2}+\phi(3)\frac{y^3}{1-y^6}+\phi(5)\frac{y^5}{1-y^{10}}+\ldots\ldots$$

$$=\frac{y}{1-y^2}+\frac{2y^3}{1-y^6}+\frac{4y^5}{1-y^{10}}+\frac{6y^7}{1-y^{14}}+\frac{8y^9}{1-y^{18}}+\ldots\ldots$$

$$=y(1+y^2+y^4+y^6+y^8+\ldots\ldots)+2y^3(1+y^6+\ldots\ldots)+4y^5(1+y^{10}+\ldots\ldots)$$

$$+6y^7(1+y^{14}+\ldots\ldots)+8y^9(1+y^{18}+\ldots\ldots)+\ldots\ldots=y+3y^3+5y^5+7y^7+9y^9+\ldots\ldots$$

$$=y(1+y^2)(1+2y^2+3y^4+4y^5+5y^8+\ldots\ldots)=y(1+y^2)(1-y^2)^{-2}=\frac{y(1+y^2)}{(1-y^2)^2}.$$

Let $y=x\surd(-1)$.

$$\therefore\phi(1)\frac{x}{1+x^2}-\phi(3)\frac{x^3}{1+x^6}+\phi(5)\frac{x^5}{1+x^{10}}-\phi(7)\frac{x^7}{1+x^{14}}+\ldots\ldots=\frac{x(1-x^2)}{(1+x^2)^2}.$$

182. Proposed by J. F. LAWRENCE, Breckenridge, Mo.

Find the values of x_1, x_2, x_3, ..., x_n which satisfy the following system of simultaneous equations:

$$\frac{x_1}{a_i-b_1}+\frac{x_2}{a_i-b_2}+\ldots\ldots+\frac{x_n}{a_i-b_n}=1\quad(i=1,\,2,\,\ldots\ldots,\,n).$$

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, West Va.

The equation $\dfrac{x_1}{\theta-b_1}+\dfrac{x_2}{\theta-b_2}+\ldots\ldots+\dfrac{x_n}{\theta-b_n}=1-\dfrac{(\theta-a_1)(\theta-a_2)\ldots\ldots(\theta-a_n)}{(\theta-b_1)(\theta-b_2)\ldots\ldots(\theta-b_n)}$,

is an equation of the $(n-1)$th degree in θ and is satisfied by the n values a_1, a_2, ..., a_n. Multiply each side by $\theta-b_1$, and then place $\theta=b_1$.

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*The general term should be considered. For $8y^9$ read $6y^9$. Editor D.

$$\therefore x_1 = -\frac{(b_1-a_1)(b_1-a_2)\ldots\ldots(b_1-a_n)}{(b_1-b_2)(b_1-b_3)\ldots\ldots(b_1-b_n)},\quad x_r = -\frac{(b_r-a_1)(b_r-a_2)\ldots\ldots(b_r-a_n)}{(b_r-b_1)(b_r-b_2)\ldots\ldots(b_r-b_n)}.$$

GEOMETRY.

203. Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, England.

Show that two parabolae can always be drawn through the vertices of a triangle to touch its circumcircle at a vertex, and that the axes of these pairs of curves are orthogonal. Show that any triangle may be circumscribed by a conic so that the tangents at each vertex are parallel to the opposite side.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, West Va.

In order that $l\beta\gamma+m\gamma\alpha+n\alpha\beta=0$ may be a parabola we must have

$$l^2a^2+m^2b^2+n^2c^2=2ablm+2bcmn+2acln.$$

For tangency at a vertex we must have $l\beta+m\alpha=0$, also $\alpha\sin B+\beta\sin A=0$.

$\therefore l/m=\sin A/\sin B=a/b$, or $l=ma/b$.

$\therefore m^2a^2/b^2+m^2b^2+n^2c^2=2a^2m^2+2bcmn+2a^2cmn/b$.

$\therefore m^2(a^2-b^2)^2+n^2b^2c^2=(2b^3c+2a^2bc)mn$.

$\therefore n=\dfrac{(a\pm b)^2}{bc}m$.

$\therefore ac\beta\gamma+bc\gamma\alpha+(a\pm b)^2\alpha\beta=0$ is the equation to the two parabolae.

If two of the sides be taken as axes, the Cartesian equation is $(x\pm y)^2+bx+ay=0$. In order that the tangents at each vertex may be parallel to the opposite sides we must have $l/m=b/a$, $m/n=c/b$.

$\therefore \beta\gamma/a+\gamma\alpha/b+\alpha\beta/c=0$ is the conic required, and this conic is an ellipse.

II. Solution by G. W. GREENWOOD, B. A., Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.

Let the triangle be formed by the lines α, β, γ. The tangent at the vertex $\alpha\beta$ to the circumcircle is

$$a\beta+b\alpha=0.$$

The equation of any circumconic having this line for a tangent may be written

$$a\beta\gamma+b\gamma\alpha+\lambda\alpha\beta=0.$$

The coördinates of the center, *i. e.*, of the pole of the line at infinity, are found to be

$$\lambda ac-a^3+ab^2,\quad \lambda bc-b^3+a^2b,\quad \lambda a^2+\lambda b^2-\lambda^2c.$$

If this point lies on the line at infinity, the conic will be a parabola. This condition gives $\lambda=(a\pm b)^2/c$, and hence there are two parabolae satisfying this condition. The coördinates of the centers become